Spin polarization of electrons in asymmetric double quantum wells under an in-plane magnetic field. Effect of abrupt barriers.

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We have studied the dependence of the electronic energy spin-splitting of $Ga_xIn_{1-x}As-Al_yIn_{1-y}As$ based double quantum wells (narrow gap structures) under in-plane magnetic fields. To do this, we have developed an improved version of the Transfer Matrix Approach that consider contributions from the abrupt interfaces and external electric fields when tunneling through central barrier exists. We have included the dependence of the Landé g-factor on the external applied field. Variations of the electron spin-splitting energy lead to some peculiarities of the density of states. Because the density of states is directly related to photoluminescence excitation, these peculiarities can be observed by this technique.

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I. INTRODUCTION

In the last decade a great interest has arisen for the so-called spintronic, or spin-based electronics. The reason is that the spintronic, due to its low power consumption, promises to be a good alternative to the traditional electronics, based on the charge transport¹. On the other hand, the spin transport is not very dissipative, with low energy losses over long distances, although the relaxation of the spin polarization may exist. Spintronics is a catchall term that refers to the potential use of the spin rather than the charge in electronic devices. For it to be useful in practice, spin up and spin down electronic states (or hole states) of any material must be separated in energy. Also, the material should be electrically polarized as in conventional electronics, which means that carriers, both negative (electrons) and positive (hole), must be able to conduct. However, except giant magnetoresistance (GMR) and its use in spin valves, spintronics is still just a promising possibility.

A key point is the choice of the material. For example, narrow gap semiconductors with strong spin-orbit coupling. GaAs-based materials seem to be the suitable candidates² due to the long life of the magnetic spin state of photoexcited electrons, which behaves coherently. It is also important that the materials have a good lattice matching to avoid defects accumulation at the interface and internal strains, which would worsen the transport of the polarized spin. There exist techniques to relax this strain, as to place step graded buffers with a progressive variation of the In concentration between the substrate and the active device. These layers mainly absorb strain and defects.

Another important point is a large Landé factor for having a significant splitting of spin states by applying small magnetic fields. In general, $In_xGa_{1-x}As - In_yAl_{1-y}As$ structure seems to be one of the most appropriate for spintronic purposes. This heterostructure offers the possibility of manipulate the gap width, the Landé factor and the interface contributions by varying x and y concentrations. Besides, it presents a remarkable spin-splitting energy when a weak magnetic fields is applied. This peculiarity makes this material suitable for high temperature spin-valves devices.

In this work we will focus in the spin-splitting changes in asymmetric double coupled quantum wells ACQW as function of an in-plane magnetic field. For this purpose we will base on an extended version of the 6×6 Kane formalism to calculate the band structure and dispersion laws³. Such electronic dispersion laws (and the corresponding spin-orbit

splitting) in quantum heterostructures have been widely studied in the last decades⁴. In theory, effects from compositional parameters and external perturbations can be analyzed through the density of states. And the modifications of the density of states can be directly observed using photoluminescence excitation technique (PLE)⁵.

In bulk materials spin-orbit interaction is caused either by a soft potential⁶ and by cubic⁷ and linear⁸ spin-dependent contributions to the effective Hamiltonian. However, in the two-dimensional (2D) case, we can reduce the cubic contribution to a linear one after the squared momentum substitution by its quantized value due to confinement⁹. Moreover, it is necessary to consider the additional spin-orbit splitting caused by the interaction with abrupt heterojunction potentials (see Ref.^{1,3,10,11}). This contribution is absolutely different to the contributions mentioned above. Splitting of 2D spin states is not feasible without the aforementioned short-range potential contribution. Unfortunately, experimental studies have not thrown enough light on induced mechanisms affecting the relative contributions of bulk and heterojunctions⁶⁻⁸.

The effect of an in-plane magnetic field on the energy spectrum in nonsymmetric heterostructures is caused by the Pauli contribution to the electron Hamiltonian. Several peculiarities for transport phenomena in heterostructures have been also discussed^{12–15}. However, these discussions only consider the mixing between Pauli contribution and effective 2D spinorbit interaction. As mentioned above, we cannot forget the contribution of the abrupt barriers in asymmetric structures, as ACQW can be, to the spin polarization. The present calculations, based on the three-band Kane model with nonsymmetric boundary conditions, try to contribute with some light to these interactions. Besides, we have included possible changes of the Landè factor. Although the Landè g-factor depends on the applied fields^{16,17}, this dependence is negligible for in-plane magnetic fields and low electron density. The model also applies to narrow-gap heterostructures whenever the slow potential generated by doping or external transverse electric fields can be described self-consistently.

II. EIGENSTATE PROBLEM

If we keep in mind the strain effect (in the case that there exists) is included in the structure through the gap, the conduction and the valence well potentials¹⁸, we will center our attention in the Hamiltonian describing the electronic behavior in the conduction band.

We will include effects of electric and magnetic fields as well as the interfaces contribution.

Based on the assumption that the gap energy ε_g is smaller than the energy distance between the valence band (v-band) and the spin-split band extrema, the electronic states in these narrow-gap heterostructures can be described by the three-band Kane matrix Hamiltonian

$$\hat{\varepsilon}(z) + (\hat{\mathbf{v}} \cdot \hat{\pi}), \qquad \hat{\pi} = \hat{\mathbf{p}} - \frac{e}{c}\mathbf{A},$$
 (1)

where the kinetic momentum, $\hat{\pi}$, contains the vector potential $\mathbf{A} = (Hz, 0, 0)$, $\mathbf{H} || OY$ is an in-plane magnetic field and $\hat{\mathbf{p}} = (\mathbf{p}, \hat{p}_z)$ is written in the \mathbf{p} , z-representation through the 2D momentum \mathbf{p} . We have also introduced the diagonal energy matrix $\hat{\varepsilon}(z)$ whose elements fix the positions of the band extrema and the interband velocity matrix $\hat{\mathbf{v}}$.

From now on it is necessary to introduce a new index $\mu = w, b$ to denote narrow-gap regions (wells) and wide-gap regions (central barrier and lateral sides), respectively. In the parabolic approximation we can write the Schrödinger equation for ACQW in the form³:

$$\left(\varepsilon_p^{\mu} + \frac{\hat{p}_z^2}{2m_{\mu}} + \varepsilon_c^{\mu}(z) + \widehat{W}^{\mu}(z)\right)\Psi^{\mu}(\mathbf{p}, z) = E\Psi^{\mu}(\mathbf{p}, z), \tag{2}$$

where the isotropic kinetic energy is given by

$$\varepsilon_p^{\mu} = \frac{p_x^2 + p_y^2}{2m_{\mu}},\tag{3}$$

which includes the effective mass m_{μ} . The $\varepsilon_c^{\mu}(z)$ energy is $\varepsilon_c^{w}(z) = U(z)$ in the wells, and $\varepsilon_c^{b}(z) = \Delta E_c + U(z)$ in the barriers, where ΔE_c is the band offset for c conduction band. Whenever energy values are less than ΔE_c underbarrier penetration (and tunneling) will be permitted and described by the boundary conditions. Now, parabolic approximation is justified because energy values under consideration are smaller than the gap energy ε_g in the narrow region. Band diagram for ACQW is shown in Fig. 1. The potential U(z), for an uniform transverse electric field, is $U(z) \simeq eF_{\perp}z$.

The magnetic energy $\widehat{W}^{\mu}(z)$, for not very strong magnetic fields, is described by

$$\widehat{W}^{\mu}(z) = -V^{\mu}(z) \left[\widehat{\boldsymbol{\sigma}} \times \mathbf{p} \right]_z + \frac{g^{\mu}(z)}{2} \mu_B H \widehat{\boldsymbol{\sigma}}_y, \tag{4}$$

where $\mu_B \equiv |e|\hbar/(m_e c)$ is the Bohr magneton and $\hat{\sigma}$ is the Pauli matrix. Finally, the characteristic spin velocity $V^{\mu}(z)$, and the effective g-factor are

$$V^{\mu}(z) = \frac{\hbar}{4m_{\mu}} \frac{d\varepsilon_c^{\mu}(z)/dz}{\varepsilon_g}, \quad g^{\mu}(z) = \frac{m_e}{2m_{\mu}} \left[1 + z \frac{d\varepsilon_c^{\mu}(z)/dz}{\varepsilon_g} \right]. \tag{5}$$

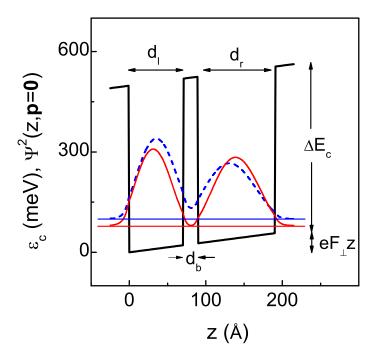


FIG. 1: (Online color) Conduction band diagram for undoped asymmetric double quantum well. Horizontal thin lines show electron energy levels and thin curves correspond to squared wave functions close to the resonance.

The potential of interfaces determines a part of the spin dependent contribution through the parameter χ . Actually, χ takes into account the spin-orbit coupling due to the abrupt potential of the heterojunction at each interface, as^{3,11}

$$\chi = \frac{2}{\hbar} \int_{-\delta}^{\delta} m_{\mu} V^{\mu}(z) dz, \tag{6}$$

where the integral is taken over the width of the abrupt interface 2δ .

In the present case, where U(z) is linear with z, we get $d\varepsilon_c^{\mu}(z)/dz = dU(z)/dz = eF_{\perp}$, which does not depend on z. Thus, the external applied electric field F_{\perp} shapes the spin velocity $V^{\mu}(z) = \overline{v}^{\mu}$ and the g-factor, $g^{\mu}(z) = g^{\mu}$. We can take the characteristic spin velocity for each layer as

$$\overline{\mathbf{v}}^{\mu} = \frac{eF_{\perp}\hbar}{4m_{\mu}\varepsilon_g},\tag{7}$$

and the abrupt interface parameter as

$$\chi = \frac{2eF_{\perp}\delta + \Delta E_c}{2\varepsilon_g} \approx \frac{\Delta E_c}{2\varepsilon_g} \tag{8}$$

Lastly, we introduce the Pauli splitting energy w_H , caused by the magnetic field, as $w_H^{\mu} = (g^{\mu}/2)\mu_B H$. Thus, Eq. (4) becomes

$$\widehat{W}^{\mu} = \bar{\mathbf{v}}^{\mu} \left[\hat{\boldsymbol{\sigma}} \times \mathbf{p} \right]_z + w_H^{\mu} \hat{\sigma}_y, \tag{9}$$

which has lost the z dependence.

Because \widehat{W}^{μ} and ε_{p}^{μ} do not depend on z, we can factorize fundamental solutions of Eq. (2), $\Psi^{\mu}(\mathbf{p},z)$, as products of $\psi^{\mu\sigma}(\mathbf{p})$ and $\varphi^{\mu\sigma}(z)$ functions, with $\sigma=\pm 1$. The σ value refers to the two possible spin orientations. For an ACQW under a transverse electric field F_{\perp} , the \mathbf{p} -dependent spinors $\psi^{\mu\sigma}(\mathbf{p})$ can be obtained from

$$\left(\varepsilon_p^{\mu} + \widehat{W}^{\mu}\right)\psi^{\mu\sigma}(\mathbf{p}) = \varepsilon_{\sigma\mathbf{p}}^{\mu}\psi^{\mu\sigma}(\mathbf{p}),\tag{10}$$

in the form

$$\psi^{\mu+}(\mathbf{p}) = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 \\ (\overline{\mathbf{v}}^{\mu} p_{+} + w_{H}^{\mu}) / i \mathbf{w}_{\mathbf{p}}^{\mu} \end{vmatrix} ,$$

$$\psi^{\mu-}(\mathbf{p}) = \frac{1}{\sqrt{2}} \begin{vmatrix} (\overline{\mathbf{v}}^{\mu} p_{-} + w_{H}^{\mu}) / i \mathbf{w}_{\mathbf{p}}^{\mu} \\ 1 \end{vmatrix} ,$$
(11)

where

$$\mathbf{w}_{\mathbf{p}}^{\mu} = \sqrt{(\bar{\mathbf{v}}^{\mu} p_x + w_H^{\mu})^2 + (\bar{\mathbf{v}}^{\mu} p_y)^2},\tag{12}$$

and the energy quasi-paraboloids are

$$\varepsilon_{\mathbf{p}\sigma}^{\mu} = \varepsilon_{p}^{\mu} + \sigma \mathbf{w}_{\mathbf{p}}^{\mu},\tag{13}$$

Now, z-dependent functions $\varphi^{\mu\sigma}(z)$ are obtained from the second order differential equation

$$\left(\frac{\hat{p}_z^2}{2m_\mu} + eF_\perp z\right) \varphi^{\mu\sigma}(z) = \left(E - \varepsilon_{\mathbf{p}\sigma}^\mu\right) \varphi^{\mu\sigma}(z).$$
(14)

For an ACQW under a transversal electric field F_{\perp} , eigenstate functions of Eq. (14) are the known linear combination of the Airy Ai- and Bi-functions. Thus, the general solution of Eq. (2) can be written as

$$\Psi^{\mu+}(\mathbf{p},z) = \psi^{\mu+}(\mathbf{p}) \left[a_{\mu+} A_i^{\mu+}(z) + b_{\mu+} B_i^{\mu+}(z) \right],
\Psi^{\mu-}(\mathbf{p},z) = \psi^{\mu-}(\mathbf{p}) \left[a_{\mu-} A_i^{\mu-}(z) + b_{\mu-} B_i^{\mu-}(z) \right],$$
(15)

where $a_{\mu\sigma}$, $b_{\mu\sigma}$ are four constants by region to be solved through the interface conditions of continuity of the wave functions and current. Because there are two wells, and three barriers (Fig. 1), there are four interfaces and four boundary conditions by interface.

Next, to simplify the calculation of the wave functions and the dispersion relations of the electronic levels through the boundary conditions^{3,10}, we create two auxiliary parameters: a length l^{μ}_{\perp} and an energy $\varepsilon^{\mu}_{\perp}$

$$l_{\perp}^{\mu} = \left(\frac{\hbar^2}{2m_{\mu}eF_{\perp}}\right)^{1/3}, \quad \varepsilon_{\perp}^{\mu} = \frac{\hbar^2}{2m_{\mu}(l_{\perp}^{\mu})^2},$$
 (16)

and a set of momentum dependent functions

$$\rho_{\mathbf{p}-}^{\mu} = \frac{p_{-}\bar{\mathbf{v}}^{\mu} + w_{H}^{\mu}}{iw_{\mathbf{p}}^{\mu}}
\rho_{\mathbf{p}+}^{\mu} = \frac{p_{+}\bar{\mathbf{v}}^{\mu} + w_{H}^{\mu}}{iw_{\mathbf{p}}^{\mu}}
f_{1\mathbf{p}}^{\mu+} = \chi \frac{\bar{\mathbf{v}}^{\mu} \left(p_{x}^{2} + p_{y}^{2}\right) + w_{H}^{\mu}p_{x}}{2\hbar w_{\mathbf{p}}^{\mu}} ,
f_{2\mathbf{p}}^{\mu+} = -\chi \frac{w_{H}^{\mu}p_{y}}{2\hbar \left(p_{+}\bar{\mathbf{v}}^{\mu} + w_{H}^{\mu}\right)} ,
f_{1\mathbf{p}}^{\mu-} = \chi \frac{w_{H}^{\mu}p_{y}}{2\hbar \left(p_{-}\bar{\mathbf{v}} - w_{H}^{\mu}\right)}
f_{2\mathbf{p}}^{\mu-} = -f_{1\mathbf{p}}^{\mu+} \tag{17}$$

where

$$p_{+} = p_{x} + ip_{y}$$
, and $p_{-} = p_{x} - ip_{y}$. (18)

Next, using preliminary quasi parabolic dispersion relations $\varepsilon_{\mathbf{p}\sigma}^{\mu}$, we construct the Airy function arguments

$$\xi_{\mathbf{p}\sigma}^{\mu} = \frac{z}{l_{\perp}^{\mu}} + \frac{\varepsilon_{\mathbf{p}\sigma}^{\mu} - E + \delta_{b}^{\mu} \Delta E_{c}}{\varepsilon_{\perp}^{\mu}},\tag{19}$$

where δ_b^{μ} acts as a Kronecker function: $\delta_b^{\mu} = 1$ when $\mu = b$, and $\delta_b^{\mu} = 0$ when $\mu = w$.

We use the transfer matrix method to obtain wave functions and energy dispersion relations of the electronic levels. In the present case, we have improved the standard method by using 4×4 matrices at each interface, whose elements are

$$\mathcal{M}_{11}^{\mu}(z, E, \mathbf{p}) = Ai\left(\xi_{\mathbf{p}+}^{\mu}\right),$$

$$\mathcal{M}_{13}^{\mu}(z, E, \mathbf{p}) = Bi(\xi_{\mathbf{p}+}^{\mu}),$$

$$\mathcal{M}_{14}^{\mu}(z, E, \mathbf{p}) = \rho_{\mathbf{p}-}^{\mu} Ai(\xi_{\mathbf{p}-}^{\mu}),$$

$$\mathcal{M}_{14}^{\mu}(z, E, \mathbf{p}) = \rho_{\mathbf{p}-}^{\mu} Bi(\xi_{\mathbf{p}-}^{\mu}),$$

$$\mathcal{M}_{21}^{\mu}(z, E, \mathbf{p}) = \rho_{\mathbf{p}+}^{\mu} Ai(\xi_{\mathbf{p}+}^{\mu}),$$

$$\mathcal{M}_{22}^{\mu}(z, E, \mathbf{p}) = \rho_{\mathbf{p}+}^{\mu} Bi(\xi_{\mathbf{p}+}^{\mu}),$$

$$\mathcal{M}_{23}^{\mu}(z, E, \mathbf{p}) = Ai(\xi_{\mathbf{p}-}^{\mu}),$$

$$\mathcal{M}_{24}^{\mu}(z, E, \mathbf{p}) = Bi(\xi_{\mathbf{p}-}^{\mu}),$$

$$\mathcal{M}_{31}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[Ai'(\xi_{\mathbf{p}+}^{\mu}) + f_{1\mathbf{p}}^{\mu+} \rho_{\mathbf{p}+}^{\mu} Ai\left(\xi_{\mathbf{p}+}^{\mu}\right) \right],$$

$$\mathcal{M}_{32}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[Bi'(\xi_{\mathbf{p}+}^{\mu}) + f_{1\mathbf{p}}^{\mu+} \rho_{\mathbf{p}+}^{\mu} Bi\left(\xi_{\mathbf{p}+}^{\mu}\right) \right],$$

$$\mathcal{M}_{33}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[\rho_{\mathbf{p}-}^{\mu} Ai'(\xi_{\mathbf{p}-}^{\mu}) + f_{2\mathbf{p}}^{\mu+} Ai(\xi_{\mathbf{p}-}^{\mu}) \right],$$

$$\mathcal{M}_{34}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[\rho_{\mathbf{p}+}^{\mu} Ai'(\xi_{\mathbf{p}+}^{\mu}) + f_{1\mathbf{p}}^{\mu-} Ai\left(\xi_{\mathbf{p}+}^{\mu}\right) \right],$$

$$\mathcal{M}_{41}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[\rho_{\mathbf{p}+}^{\mu} Ai'(\xi_{\mathbf{p}+}^{\mu}) + f_{1\mathbf{p}}^{\mu-} Ai\left(\xi_{\mathbf{p}+}^{\mu}\right) \right],$$

$$\mathcal{M}_{42}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[Ai'(\xi_{\mathbf{p}-}^{\mu}) + f_{2\mathbf{p}}^{\mu-} \rho_{\mathbf{p}-}^{\mu} Ai\left(\xi_{\mathbf{p}-}^{\mu}\right) \right],$$

$$\mathcal{M}_{43}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[Ai'(\xi_{\mathbf{p}-}^{\mu}) + f_{2\mathbf{p}}^{\mu-} \rho_{\mathbf{p}-}^{\mu} Ai\left(\xi_{\mathbf{p}-}^{\mu}\right) \right],$$

$$\mathcal{M}_{44}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[Ai'(\xi_{\mathbf{p}-}^{\mu}) + f_{2\mathbf{p}}^{\mu-} \rho_{\mathbf{p}-}^{\mu} Ai\left(\xi_{\mathbf{p}-}^{\mu}\right) \right],$$

$$\mathcal{M}_{44}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[Ai'(\xi_{\mathbf{p}-}^{\mu}) + f_{2\mathbf{p}}^{\mu-} \rho_{\mathbf{p}-}^{\mu} Ai\left(\xi_{\mathbf{p}-}^{\mu}\right) \right],$$

$$\mathcal{M}_{44}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[Ai'(\xi_{\mathbf{p}-}^{\mu}) + f_{2\mathbf{p}}^{\mu-} \rho_{\mathbf{p}-}^{\mu} Ai\left(\xi_{\mathbf{p}-}^{\mu}\right) \right],$$

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$$\mathcal{M}_{44}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[Ai'(\xi_{\mathbf{p}-}^{\mu}) + f_{2\mathbf{p}}^{\mu-} \rho_{\mathbf{p}-}^{\mu} Ai\left(\xi_{\mathbf{p}-}^{\mu}\right) \right],$$

$$\mathcal{M}_{45}^{\mu}(z, E, \mathbf{p}) = \frac{m_e}{m_{\mu}} \left[Ai'(\xi_{\mathbf{p}-}^{\mu}) + f_{2\mathbf{p}}^{\mu-} \rho_{\mathbf{p}-}^{\mu} Ai\left(\xi_{\mathbf{p}-}^{\mu}\right) \right],$$

where $Ai'(\xi_{\sigma \mathbf{p}}^{\mu})$ means $dAi(\xi_{\sigma \mathbf{p}}^{\mu})/dz$ and $Bi'(\xi_{\sigma \mathbf{p}}^{\mu})$ means $dBi(\xi_{\sigma \mathbf{p}}^{\mu})/dz$. Now we are ready to generate transfer matrices, $\widetilde{S}^{\mu}(z, E, \mathbf{p}) = \mathcal{M}^{\mu}_{ij}(z, E, \mathbf{p})$. Finally, electronic levels for each 2D momentum $\mathbf{p} = (p_x, p_y)$ are obtained from $\widetilde{S}_{44}(E, \mathbf{p}) = 0$, where

$$\widetilde{S}(E, \mathbf{p}) = \left[\widetilde{S}^{b}(L_{0}, E, \mathbf{p})\right]^{-1} \cdot \widetilde{S}^{w}(L_{0}, E, \mathbf{p}) \cdot \left[\widetilde{S}^{w}(L_{1}, E, \mathbf{p})\right]^{-1} \cdot \widetilde{S}^{b}(L_{1}, E, \mathbf{p}) \cdot \left[\widetilde{S}^{b}(L_{2}, E, \mathbf{p})\right]^{-1} \cdot \widetilde{S}^{w}(L_{2}, E, \mathbf{p}) \cdot \left[\widetilde{S}^{w}(L_{3}, E, \mathbf{p})\right]^{-1} \cdot \left[\widetilde{S}^{b}(L_{3}, E, \mathbf{p})\right].$$

$$(21)$$

In the above matrix product, $z = L_i$ denotes interfaces position in the growth direction, starting from the left side.

Calculations give us the two coupled spin up paraboloids $E_{k+}(\mathbf{p})$ and spin down paraboloids $E_{k-}(\mathbf{p})$, where k=1,2 corresponds to the deepest levels of the ACQW. Once obtained the involved coefficients $a_{\mu\sigma}$, $b_{\mu\sigma}$ we normalize wave functions for each momentum \mathbf{p} .

The scheme of Fig. 1 includes the two resonant energy levels and the respective wave functions for $\mathbf{p} = \mathbf{0}$. Although there are four levels only two are observable in this figure. This is because spin sublevel splitting is much smaller than electronic level energy distance and differences between spin down and spin up wave functions are not visible at $\mathbf{p} = \mathbf{0}$.

Finally, the density of states can be obtained by using the well-known expression

$$\rho(\varepsilon) = \sum_{k,\sigma} \int \frac{d\mathbf{p}}{(2\pi\hbar)^2} \delta(\varepsilon - E_{k\sigma}(\mathbf{p})). \tag{22}$$

Peculiarities of $\rho(\varepsilon)$ can be analyzed through the photoluminescence excitation (PLE) intensity for the case of near-edge absorption, I_{PLE} , because both quantities are related by ^{9,19}

$$I_{PLE} \sim \sum_{\lambda_c \lambda_v} |\mathbf{e} \cdot \mathbf{v}_{cv}|^2 \delta(\varepsilon_{\lambda_c} - \varepsilon_{\lambda_v} - \hbar\omega) \sim \rho(\hbar\Delta\omega)$$
 (23)

for very low temperature or

$$I_{PLE} \sim \sum_{\lambda_c \lambda_c} |\mathbf{e} \cdot \mathbf{v}_{cv}|^2 G(\varepsilon_{\lambda_c} - \varepsilon_{\lambda_v} - \hbar \omega),$$
 (24)

when one includes electron-phonon scattering. In the above expression, the Gaussian function is $G(x,x_0)=\frac{1}{\gamma\sqrt{2\pi}}\exp\left[-\left(\frac{x-x_0}{\gamma\sqrt{2}}\right)^2\right]$. The Gaussian halfwidth γ is related to the scattering and relaxation processes and, thus, to the temperature²⁰. Expressions (23, 24) are valid provided the interband velocity \mathbf{v}_{cv} does not depend on in-plane momentum. Here \mathbf{e} is the light polarization vector, $\Delta\omega=\omega-\varepsilon_g/\hbar$, and ε_{λ_c} , ε_{λ_v} are the conduction and valence band levels, respectively.

III. RESULTS

Let's start this section with numerical results for $In_xGa_{1-x}As - In_yAl_{1-y}As$ -based AC-QWs, with x = 0.53 and y = 0.52. We have chosen this particular structure because we have reliable data for basic parameters¹⁹. We have considered two InGaAs wells of 70 and 100 Å wide separated by a 20 Å InAlAs barrier. We have also applied an electric field of 30

kV/cm, which corresponds to a spin velocity $\overline{v}^w = 2.6 \times 10^5$ cm/s for the InGaAs QWs, and $\overline{v}^b = 1.4 \times 10^5$ cm/s for the InAlAs barriers, with a transition spin velocity region across de abrupt interface. This electric field is slightly higher than needed to achieve resonance between the deepest levels of both wells (28 kV/cm). To calculate interface contributions we have used a typical abrupt interface size of $\delta \sim 3\mathring{A}$ for $InAlAs - InGaAs^{21}$. We have also applied in-plane magnetic field of 0.01 T.

Figures 2(a-d) show normalized squared wave function for $p_y=0$, versus z and the dimensionless momentum p_x/p_0 , where $p_0 = m_w \overline{\mathbf{v}}^w$. Upper panels (a,b) correspond to the first deepest level for the two different spin orientations. As expected for an electric field beyond the resonance, charge density is mainly located in the left narrow QW. Consequently, the opposed happens for the second resonant level as can be seen in the lower panels (c,d)Analyzing the behavior of wave functions versus momentum p_x and spin orientations by comparing panels (a) and (b), a new peculiarity appears. While the charge distribution coincide for both down and up spins at the zone center $(p_x = 0)$, there is a charge transfer between wells for increasing $|p_x|$. For spin down case [panel (a)] charge experiences a little shift from left narrow well to the right wide one. As a result, to conserve the charge, the opposite occurs for spin up electrons [panel (b)] where charge goes from the right to the left well. For the higher resonant level we find a misleading ambiguity. It might seem the behavior is the opposite to the previous one because now, the spin down case [panel (c)] shows a charge transfer from right to left QW as $|p_x|$ increases and, conversely, for spin up electrons [panel (d)]. However, considering the relative charge concentration between wells, we can realize there is a similar behavior for both resonant levels. For spin down electrons there is a charge shift from well with higher concentration to the other well [panels (a), (c)] and conversely for the spin up electrons [panels (b), (d)]. The reason for this charge shift lies in the energy term $w_{\mathbf{p}}^{\mu}$, which induces a breaking of the momentum symmetry. Because of $w_{\mathbf{p}}^{\mu}$ is an essential part of the argument of the Airy functions, the behavior of the wave functions is significantly affected.

Fig. 3 shows the near parabolic dispersion relations of the two coupled levels and their corresponding spin up and down sublevels, for the electric and magnetic fields under consideration. It can be seen the p_y paraboloids symmetry agrees with Eqs. (11) and (18). Although there is a little difference between spin paraboloids, due to the large energy difference between the resonant levels of both wells (\sim 12 meV) and the splitting of the spin

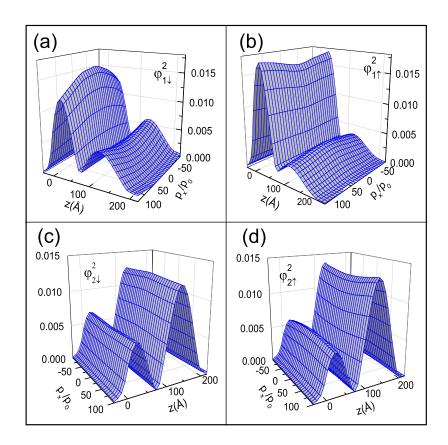


FIG. 2: (Online color) Squared wave function vs normalized p_x/p_0 for the first spin resonant levels.

sublevels (~ 0.01 meV at $p_y = 0$) it is not possible to distinguish minima behavior. Thus, we have enlarged in Fig. 4 the bottom of the pair of paraboloids (spin up and spin down) for the ground level. After resonance this level would be associated to the deepest level of the left narrow QW, if wells were decoupled. As expected, both paraboloids shift in opposite p_x directions resulting in a sublevel anticrossing.

Considering a $p_y = 0$ section of the former figure we can get a more accurate 2D representation (Fig. 5) of the anticrossing, minima p_x position, and energy splitting. The inset displays anticrossing area enlargement. In order to have a more detailed overview of the anticrossing region we have also included in Fig. 6 the contour plot around anticrossing for different constant energy values.

Next, we analyze the density of states $\rho(\varepsilon)$ because this function is proportional to

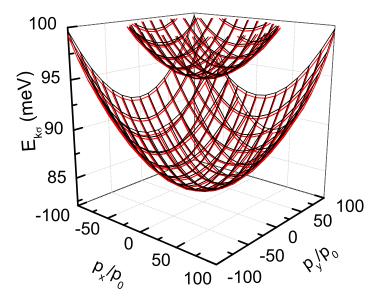


FIG. 3: (Online color). Near parabolic dispersion relations of the two coupled levels close to the resonance, and their corresponding spin up and spin down sublevels.

the photoluminescence excitation (PLE) intensity²². And PLE is one of the most used techniques to get information of quantum structures. In this case we have used magnetic field intensities varying from 0 to 0.1 T. The shape of $\rho(\varepsilon)$, shown in Fig. 7, is characterized for the peaks coming from he $\varepsilon^{-1/2}$ -divergence corresponding to the zero-field case. As expected, these peaks disappear gradually by growing magnetic field. In turn, interfaces contribute with a manifested delay in the quenching of the $\rho(\varepsilon)$ singularities, as well as an additional broadening of these peaks. That is, although the singularities should only appear at zero magnetic field, they still remain at the band anticrossing position for low magnetic fields, as shown in Fig. 7. Another significant feature is that the $\rho(\varepsilon)$ anticrossing peak is softened when increasing barrier height, disappearing for lower magnetic fields than used in this work³

Finally, Fig. 8 shows PLE spectra for different Gaussian halfwidth γ at a fixed H=0.1 T. For the magnetic field used before (H=0.01 T) the two adjacent peaks, corresponding to each resonant pair of states, overlap. Thus, we have used a magnetic field ten times higher because this field allows us to tell the peaks apart for small γ values. Evolution of the first

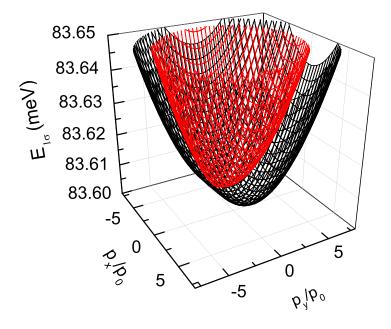


FIG. 4: (Online color) Bottom of the two deepest paraboloids just after resonance and detail of spin up and spin down anticrossing. Levels are mainly located in the narrow (left) QW.

two peaks with γ is depicted in Fig. 9. As can be seen, PLE peaks corresponding to the two different spin transitions are still distinguishable for γ values beyond 1 meV.

IV. CONCLUSIONS

In this paper we have analyzed the electron spin behavior in narrow-gap ACQWs under transverse electric and in-plane magnetic fields. We used the Kane model with nonsymmetric boundary conditions, caused by the two different wells width, to solve the eigenvalue problem. Based in this model and the transfer matrix approach we have performed a useful tool to tackle any layered structure with abrupt interfaces and subjected to different perturbations. The model allows us the study of the spin peculiarities of level anticrossings.

Because interface contributions oppose intrinsic spin-orbit effect, mechanisms that mix the Pauli contribution with the two kinds of spin-orbit contributions (from a low magnetic field and from heterojunctions) are different. As a result numerical calculations lead to magnetoinduced variations of the energy spectra under in-plane magnetic field strength

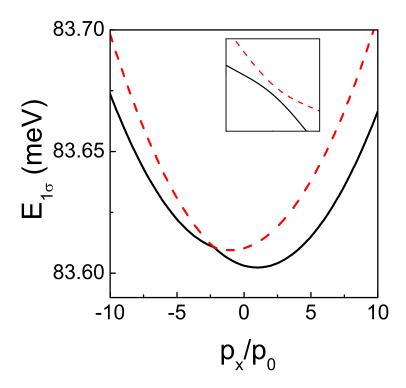


FIG. 5: (Online color) Two dimensional dispersion relations for $p_y/p_0 = 0$. Inset shows an magnified image of the spin up and spin down anticrossing.

even lower than Tesla for typical parameters of InGaAs/InAlAs structures.

There are essentially two techniques to check electron energy spectrum changes obtained in the present work. One of them, the magnetotransport measurements of Shubnikov-de Haas oscillations under nearly in-plane magnetic fields^{13,14} provide us information of in-plane magnetoinduced modifications of electron energy. The other technique to find characteristics of energy spectrum is the mid-infrared PLE spectroscopy²². This technique is suitable because it provides direct information of the energy spectrum when interband transitions are modified by in-plane magnetic fields. Moreover, PLE intensity is directly related to the density of states, which we can evaluate with the present model from the energy dispersion relations.

A detailed analysis beyond the Kane model does not modify essentially our formulation of the eigenstate problem. Thus, the main conclusions of the paper on the changes of slow-

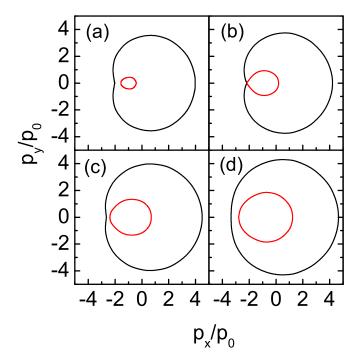


FIG. 6: (Online color). Contour plot of the deepest energy levels around anticrossing. (a) $E_{1\sigma} = 83.6100 \text{ meV}$; (b) $E_{1\sigma} = 83.611 \text{ meV}$; (c) $E_{1\sigma} = 83.612 \text{ meV}$; and (d) $E_{1\sigma} = 83.614 \text{ meV}$.

electric field and heterojunction-induced spin-splitting mechanisms under in-plane magnetic field for electron spin tunneling between wells in ACQWs, remain still valid. We have also used along the work the assumption that the confined potential is well described. Thus, the effective transverse field F_{\perp} provides correct estimations both for magnetoinduced changes and for the character of the dispersion laws. However, more detailed numerical calculations are needed to describe the kinetic behavior mentioned above. We will return to this point in a forthcoming work where we will analyze the spin dynamics in ACQWs. In summary, we have shown that essential variations of the dispersion relations in ACQWs occurs when we include contribution of the abrupt barriers. Electron density of states and PLE reflect these modifications in narrow-gap structures under in-plane magnetic fields when tunneling

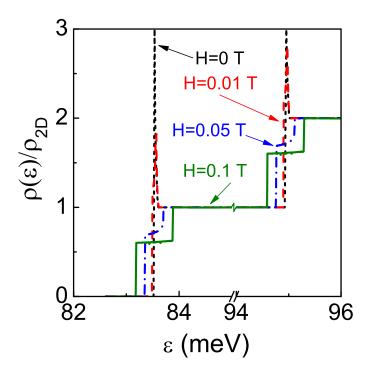


FIG. 7: (Online color) Density of states for the same electric field of previous figures and different magnetic fields.

between wells occurs.

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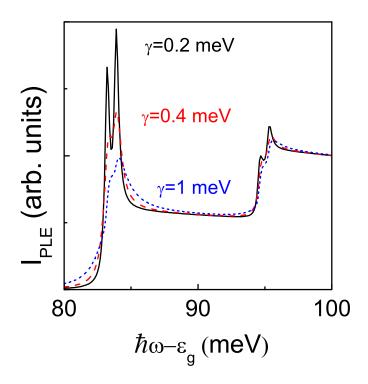


FIG. 8: (Online color) Photoluminescence intensity for $H=0.1~\mathrm{T}$ and different γ values.

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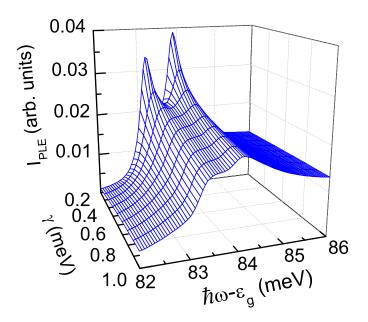


FIG. 9: (Online color) Photoluminescence behaviour vs γ corresponding to the two first transitions. Parameters are the same as in Fig. 8.

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